

## CLAIMS

The embodiments of the invention in which an exclusive property or privilege is claimed are defined as follows:

1-20 (previously cancelled)

21-29 (cancelled)

30. (New) A method of forming/defining and solving a model of <sup>the</sup> power network to <sup>affect</sup> ~~effect~~ control of voltages and power flows in a power system, comprising the steps of:
- obtaining on-line/simulated data of open/close status of switches and circuit breakers in the power network, and reading data of operating limits of <sup>the</sup> network components including PV-node, a generator-node where Real-Power-P and Voltage-Magnitude-V are given/assigned/specified/set, generators maximum and minimum reactive power generation capability limits and transformers tap position limits,
  - obtaining on-line readings of given/assigned/specified/set real and reactive power at PQ-nodes, the load-nodes where Real-Power-P and Reactive-Power-Q are given/assigned/specified/set, real power and voltage magnitude at PV-nodes, voltage magnitude and angle at the reference/slack node, and transformer turns ratios, which are the controlled variables/parameters,
  - initiating loadflow calculation with initial approximate/guess solution of the same voltage magnitude and angle as those of the (slack/reference) node for all the other nodes referred to as the slack-start, <sup>reference/slack</sup>
  - forming and storing factorized gain matrices [Y $\theta$ ] and [YV] using the same indexing and addressing information for both as they are of the same dimension and sparsity structure, wherein said [Y $\theta$ ] relate vector of modified real power mismatches [RP] to angle corrections vector [ $\Delta\theta$ ] in equation [RP] = [Y $\theta$ ] [ $\Delta\theta$ ] referred to as P- $\theta$  sub-problem, and said [YV] relate vector of modified reactive power mismatches [RQ] to voltage magnitude corrections vector [ $\Delta V$ ] in equation [RQ] = [YV] [ $\Delta V$ ] referred to as Q-V sub-problem,
  - restricting transformation/rotation angle  $\Phi_p$  to maximum  $-48^\circ$  in determining transformed real and reactive power mismatch as,

performing loadflow calculation by solving a Super Super Decoupled Loadflow model of the power network defined by set of equations  $[RP] = [Y\theta] [\Delta\theta]$  and  $[RQ] = [YV] [\Delta V]$  employing successive (10, 1V) iteration scheme, wherein each iteration involves one calculation of  $[RP]$  and  $[\Delta\theta]$  to update voltage angle vector  $[\theta]$  and then one calculation of  $[RQ]$  and  $[\Delta V]$  to update voltage magnitude vector  $[V]$ , to calculate values of the voltage angle and the voltage magnitude at PQ-nodes, voltage angle and reactive power generation at PV-nodes, and turns ratio of tap-changing transformers in dependence on the set of said obtained-online readings, or given/scheduled/specified/set values of controlled variables/parameters and physical limits of operation of the network components,

evaluating loadflow calculation for any of the over loaded power network components and for under/over voltage at any of the network nodes,

correcting one or more controlled parameters and repeating the calculating, performing, evaluating, and correcting steps until evaluating step finds no over loaded components and no under/over voltages in <sup>the</sup> power network, and

<sup>attaching</sup> ~~effecting~~ a change in the power flowing through network components and voltage magnitudes and angles at the nodes of <sup>the</sup> power network by actually implementing the finally obtained values of controlled variables/parameters after evaluating step finds a good power system or alternatively a power network without any overloaded components and under/over voltages, which finally obtained controlled variables/parameters however are stored in case of simulation for acting upon fast in case the simulated event actually occurs.

31. (New) A method as defined in claim-21 wherein loadflow calculation involving formation and solution of super super decoupled loadflow model, employing simultaneous (1V, 10) iteration scheme is characterized in that it involve only one time calculation of real and reactive power mismatches in an iteration along with modified real power mismatch calculation, depending on super super decoupled loadflow model used, either by:

$$RP_p = [(\Delta P_p / V_p) - \sum_{q=1}^m G_{pq} \Delta V_q] / V_p \quad \text{-for all nodes} \quad \text{or} \quad (74)$$

TIC-STH 2009

# Super Super Decoupled Loadflow

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**Abstract** – A generalized Super Super decoupled Loadflow referred to as SSDL-X'X' method is presented along with its special case manifestations leading to the proposed SSDL-YY method. Gain matrices of the SSDL-YY method are defined independent of rotation angles and they differ only in their diagonal terms. The diagonal terms in the gain matrix of Q-V sub-problem are characterized by the presence of transformed specified/known quantities. Rotation angles are limited to the maximum value of  $-48^\circ$ , and the slack-start as a guess solution is suggested for increased efficiency. These along with simple modification of real power mismatches at PV-nodes yields a robust and fast loadflow method. An advanced version of the General-purpose Fast Decoupled Loadflow (GFDL) is manifested as SSDL-BX method. The proposed method is tested and compared with Fast Super Decoupled Loadflow (FSDL), SSDL-BX, and GFDL methods under wide range of simulated system operating conditions and network parameters. The proposed method is superior to the FSDL method under all most all conditions and can be more than twice faster than the SSDL-BX, and GFDL methods for systems having large R/X ratio branches.

**Indexing terms:** Loadflow, Transformation, Decoupling

## I. NOMENCLATURE

$Y_{pq} = Y_{pq} \angle (\Phi_{pq} + 180^\circ) = G_{pq} + jB_{pq}$  : (p-q) th element of nodal admittance matrix without shunts  
 $y_p = y_p \angle (\Phi_{ph}) = g_p + jb_p$  : total shunt admittance at any node-p  
 $V_p = e_p + jf_p = V_p \angle \theta_p$  : complex voltage of any node-p  
 $V_s = e_s + jf_s = V_s \angle \theta_s$  : complex slack-node voltage  
 $\Delta \theta_p, \Delta V_p$  : voltage angle, magnitude corrections  
 $P_p + jQ_p$  : net nodal injected power, calculated  
 $\Delta P_p + j\Delta Q_p$  : nodal power residue or mismatch  
 $RP_p + jRQ_p$  : modified nodal power residue or mismatch  
 $PSH_p + jQSH_p$  : net nodal injected power, scheduled/specified  
 $C_p = 1 \angle \Phi_p = \cos \Phi_p + j \sin \Phi_p$  : Unitary rotation/transformation  
 $m$  : number of PQ-nodes  
 $k$  : number of PV-nodes  
 $n = m + k + 1$  : total number of nodes  
 $q > p$  : node-q is connected to node-p excluding the case of  $q = p$

## II. INTRODUCTION

Loadflow, in power system studies, is the most basic frequently performed steady state analysis of an electrical power network. Loadflows are performed in system planning, operational planning, and operation control. Of the various methods proposed including the first time introduced in [1] decoupled loadflow referred to as Voltage Vector Method (VVM), the Fast Decoupled Loadflow (FDL) [2] method was the state-of-the-practice because of its simplicity, relatively high reliability, and computational efficiency. However, it is known to suffer poor convergence for systems having high R/X ratio branches. The General-purpose version of the Fast Decoupled Loadflow (GFDL) [3] is a simple modification of

the FDL with much improved convergence in the presence of high R/X ratio branches. However, it has been shown to have failed to converge to a solution when FDL converged [3].

The transformation or super decoupled approach [4], [5] is mathematically sound and therefore, more general and reliable. Rotation operators applied to the complex node injections and corresponding admittance values that relate them to the system state variables, transform the network equations such that branch admittances appear to be almost entirely reactive. Better decoupling is thus realized. The initial methods of [4] and [5] were only slightly better convergent than those of [2] and [3]. However, the super decoupling technique with gain matrices defined independently of rotation angles, restriction of rotation angle to maximum of  $-36^\circ$  and a simple modification of real power mismatches at PV-nodes led to the development of Fast Super Decoupled Loadflow (FSDL) [6] method. The process that began with the VVM leading to the state-of-the-practice FDL method and then to the current state-of-the-art FSDL method is being taken to the new state-of-the-art proposed Super Super Decoupled Loadflow (SSDL-YY) method in this paper.

### A. Decoupled Loadflow

In a class of decoupled loadflow models, each decoupled loadflow model comprises a system of equations (1) and (2) differing in the definition of elements of [RP], [RQ], and [Yθ] and [YV]. It is a system of equations for the separate calculation of voltage angle and voltage magnitude corrections.

$$[RP] = [Y\theta] [\Delta\theta] \quad (1)$$

$$[RQ] = [YV] [\Delta V] \quad (2)$$

### B. Successive (1θ, 1V) Iteration Scheme

In this scheme (1) and (2) are solved alternately with intermediate updating. Each iteration involves one calculation of [RP] and [Δθ] to update [θ] and then one calculation of [RQ] and [ΔV] to update [V]. The sequence of relations (3) to (6) depicts the scheme.

$$[\Delta\theta] = [Y\theta]^{-1} [RP] \quad (3)$$

$$[\theta] = [\theta] + [\Delta\theta] \quad (4)$$

$$[\Delta V] = [YV]^{-1} [RQ] \quad (5)$$

$$[V] = [V] + [\Delta V] \quad (6)$$

The scheme involves solution of system of equations (1) and (2) in an iterative manner depicted in the sequence of relations (3) to (6). This scheme requires mismatch calculation for each half iteration because [RP] and [RQ] is calculated always using the most recent voltage angle and voltage magnitude values, and it is block Gauss-Seidel approach. The scheme is block successive, which imparts increased stability to the solution process, and it in turn improves convergence and increases the reliability of obtaining solution.

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### III. THE PROPOSED MODEL

It can be seen from the Jacobian elements determined at the slack-start for formulations of the power flow equations in the appendix-A that Jacobian is symmetrical for  $I=0$ , and unsymmetrical for all other values of raised power "I". Phase shifters are assumed absent and shunt conductances are ignored for the Jacobian to be symmetrical in the case of formulation of the power flow equations in appendix-A for  $I=0$ , for which the constant matrix model is given in [6]. Coupling between the active and reactive power flows in a power circuit is mainly attributed to the resistive component of the impedance. Decoupled loadflow models consequently have solution convergence problems in the presence of transmission lines of large R/X ratios. Better decoupling of the loadflow equations for networks having branch elements of high R/X is achieved through simple linear transformation of complex power injections as given in appendix-9.2 of [6].

#### A. Super Super Decoupled Loadflow: Version: SSDL-X'X'

Transformed power flow equations (42) and (43) in appendix-B are of the same form as the original real and reactive power flow equations contained in (32) and (33) in appendix-A. The only difference is that they involve conductances and susceptances values of the rotated/transformed branch admittances. The most general Super Super Decoupled Loadflow model can be realized as SSDL-X'X', from which many versions are derived. The elements of [RP], [RQ], [Yθ] and [YV] constituting the model SSDL-X'X' are defined by (7) to (19).

$$RP_p = [\Delta P_p' + (G_{pp}'/B_{pp}') \Delta Q_p'] / V_p^2 \quad \text{for PQ-nodes} \quad (7)$$

$$RQ_p = [\Delta Q_p' - (G_{pp}'/B_{pp}') \Delta P_p'] / V_p \quad \text{for PQ-nodes} \quad (8)$$

$$RP_p = [\Delta P_p' / (K_p V_p^2)] \quad \text{for PV-nodes} \quad (9)$$

$$Y\theta_{pq} = -1/X_{pq}' \quad \text{and} \quad YV_{pq} = -1/X_{pq}' \quad (10)$$

$$Y\theta_{pp} = \sum_{q \neq p} Y\theta_{pq} \quad \text{and} \quad YV_{pp} = b_p' + \sum_{q \neq p} YV_{pq} \quad (11)$$

Where,

$$b_p' = 1/[QSH_p' - (G_{pp}'/B_{pp}')PSH_p'] / V_p^2 - (2 - I) b_p \cos \Phi_p \quad (12)$$

$$\Delta P_p' = \Delta P_p \cos \Phi_p + \Delta Q_p \sin \Phi_p \quad \text{for PQ-nodes} \quad (13)$$

$$\Delta Q_p' = \Delta Q_p \cos \Phi_p - \Delta P_p \sin \Phi_p \quad \text{for PQ-nodes} \quad (14)$$

$$PSH_p' = PSH_p \cos \Phi_p + QSH_p \sin \Phi_p \quad \text{for PQ-nodes} \quad (15)$$

$$QSH_p' = QSH_p \cos \Phi_p - PSH_p \sin \Phi_p \quad \text{for PQ-nodes} \quad (16)$$

$$\cos \Phi_p = | [B_{pp}' / \sqrt{(G_{pp}'^2 + B_{pp}'^2)} ] | \geq \cos (0^\circ \text{ to } -90^\circ) \quad (17)$$

$$\sin \Phi_p = - | [G_{pp}' / \sqrt{(G_{pp}'^2 + B_{pp}'^2)} ] | \geq \sin (0^\circ \text{ to } -90^\circ) \quad (18)$$

$$K_p = | (B_{pp}' / Y\theta_{pp}) | \quad (19)$$

Where,  $G_{pp}'$  and  $B_{pp}'$  are the transformed values of  $G_{pp}$  and  $B_{pp}$  as given in the appendix-B by (45) and used in (52) in the appendix-C. The factor  $K_p$  of (19), determined experimentally; is restricted to the minimum value of 0.75, but its restriction is reduced to the minimum 0.6 if its average over all less than 1.0 values at PV-nodes is less than 0.6. This factor is system and model independent, and does not require tuning. The factor "I" in (12) can take any value from  $I = -\infty, \dots, -2, -1, 0, 1, 2, \dots$ ,

$+\infty$ . However, it has been determined experimentally that  $I = 1$  gives the best possible convergence for all systems and it is also system and model independent. But in situations where the diagonal dominance in [YV] is influenced in power networks having capacitive series branches and/or excessive shunt capacitive compensation deteriorating convergence and reliability of obtaining solution, the best possible convergence can be obtained by tuning the factor "I" in (12).

The definition of  $Y\theta_{pq}$  in (10) is simplified because it does not explicitly state that it always takes the value of  $-B_{pq}$  for a branch connected between two PV-nodes or a PV-node and the slack-node. This fact should be understood implied in all the definitions of  $Y\theta_{pq}$  in this document.  $X_{pq}'$  is the transformed branch reactance defined in the appendix-C by (51).

#### B. Super Super Decoupled Loadflow: Version: SSDL-YY

If unrestricted rotation is applied to complex branch admittance and transformed susceptance is taken as admittance magnitude value with the same algebraic sign and transformed conductance is assumed zero in (44) of Appendix-B or (50) of Appendix-C, the SSDL-X'X' model reduces to SSDL-YY. Though, this model is not very sensitive to the restriction applied to nodal rotation angles, SSDL-YY presented here restricts rotation angles to the maximum of  $-48^\circ$  in (17) and (18) determined experimentally for the best possible convergence from non-linearity considerations. However, it gives closely similar performance over wide range of restriction applied to the nodal rotation angles say, from  $-36^\circ$  to  $-90^\circ$ .

$$RP_p = (\Delta P_p \cos \Phi_p + \Delta Q_p \sin \Phi_p) / V_p^2 \quad \text{for PQ-nodes} \quad (20)$$

$$RQ_p = (\Delta Q_p \cos \Phi_p - \Delta P_p \sin \Phi_p) / V_p \quad \text{for PQ-nodes} \quad (21)$$

$$Y\theta_{pq} = \begin{cases} -Y_{pq} & \text{for branch r/x ratio} \leq 3.0 \\ -(B_{pq} + 0.9(Y_{pq} - B_{pq})) & \text{for branch r/x ratio} > 3.0 \\ -B_{pq} & \text{for branches connected between two PV-nodes or a PV-node and the slack-node} \end{cases} \quad (22)$$

$$YV_{pq} = \begin{cases} -Y_{pq} & \text{for branch r/x ratio} \leq 3.0 \\ -(B_{pq} + 0.9(Y_{pq} - B_{pq})) & \text{for branch r/x ratio} > 3.0 \end{cases} \quad (23)$$

Where,  $b_p'$  of (12) becomes,

$$b_p' = 1/(QSH_p \cos \Phi_p - PSH_p \sin \Phi_p) / V_p^2 - (2 - I) b_p \cos \Phi_p \quad (24)$$

Branch admittance magnitude in (22) and (23) is of the same algebraic sign as its susceptance. Elements of the two gain matrices differ in that diagonal elements of [YV] additionally contain the  $b_p'$  values given by (11) and (24) and in respect of elements corresponding to branches connected between two PV-nodes or a PV-node and the slack-node. The model consists of (3) to (6), (20) to (24), (9), (11), (17), (18), and (19) with rotation angles restricted to the maximum of  $-48^\circ$  in (17) and (18). In case of systems of only PQ-nodes and without any PV-nodes, equations (22) and (23) simply be taken as  $Y\theta_{pq} = YV_{pq} = -Y_{pq}$ . In two simple variations of the SSDL-YY model, one is to make  $YV_{pq} = Y\theta_{pq}$  and the other is to make  $Y\theta_{pq} = YV_{pq}$ .

#### C. Super Super Decoupled Loadflow: Version: SSDL-XX

If no or zero rotation ( $\Phi_p = 0^\circ$ ) is applied, the SSDL-X'X' model reduces to SSDL-XX, which is the simplest form of SSDL-X'X'. The SSDL-XX model comprises (3) to (6), (25) to (29), (9), (11), and (19).

$$RP_p = [\Delta P_p + (G_{pp}/B_{pp}) \Delta Q_p]/V_p^2 \quad \text{:for PQ-nodes} \quad (25)$$

$$RQ_p = [\Delta Q_p + (G_{pp}/B_{pp}) \Delta P_p]/V_p \quad \text{:for PQ-nodes} \quad (26)$$

$$Y\theta_{pq} = \begin{cases} -1.0/X_{pq} & \text{:for all other branches} \\ -B_{pq} & \text{:for branches connected between two PV-nodes or a PV-node and the slack-node} \end{cases} \quad (27)$$

$$YV_{pq} = -1.0/X_{pq} \quad \text{:for all branches} \quad (28)$$

Where,  $b_p'$  of (12) becomes,

$$b_p' = 1 [QSH_p - (G_{pp}/B_{pp}) PSH_p]/V_s^2 - (2 - T) b_p \quad (29)$$

This is the simplest model with very good performance for distribution networks containing only PQ-nodes. The large value of the difference  $[(1/X_{pq}) - B_{pq}]$ , particularly for high R/X ratio branches connected between two PV-nodes or a PV-node and the slack-node, creates modeling error in a symmetrical gain matrix model of a power network containing PV-nodes.

#### D. Super Super Decoupled Loadflow: Version: SSDL-BX

If super decoupling is applied only to QV-sub problem, the SSDL-XX model reduces to SSDL-BX, which makes it perform better for systems containing PV-nodes. The SSDL-BX model comprises (3) to (6), (26), (30), (31), (11) and (29).

$$RP_p = \Delta P_p/V_p^2 \quad \text{:for all-nodes} \quad (30)$$

$$Y\theta_{pq} = -B_{pq} \quad \text{and} \quad YV_{pq} = -1/X_{pq} \quad (31)$$

It should be noted that Amerongen's General-purpose Fast Decoupled Loadflow model [3] has turned out to be an approximation of this model. The approximation involved is only in (26) and (29). However, numerical performance is found to be better and more reliable than that of the Amerongen's method. This model can be referred to as advanced GFDL model.

#### E. Slack-start

The same voltage magnitude and angle as those of the slack-node when used for all nodes as initial guess solution, provide Jacobian elements determined in the appendix-A. With the specified magnitudes, PV-nodes voltage magnitudes are adjusted to their known values after the first half iteration. This start procedure referred to as the slack-start, saves almost all effort of mismatch calculation in the first P- $\theta$  iteration as it requires only shunt flows to be calculated at each node.

#### F. Model involving Global Corrections

The non-linearity retaining technique [7] or an exact second order model [8] formulations for the solution of the loadflow problem in rectangular coordinates produce corrections to the initial guess solution in each iteration. However, the use of the corrections to the initial solution estimate in each iteration is reported first in [1] for the loadflow problem formulation in polar coordinates. A generalized technique for the formulation

of all the loadflow models that produces corrections to initial guess solution in each iteration is given in appendix-D. It involves storage of the modified mismatch of the current iteration for adding into the modified mismatch calculated in the next iteration.

#### G. Sequence of steps for the solution of SSDL-YY model

All possible variations of the Super Super Decoupled Loadflow models are derived from the most general model SSDL-X'X' in the above and in [18]. It has been verified that SSDL-YY version is the simplest possible general purpose model that can perform well for power networks containing PV-nodes as well as those that contain only PQ-nodes. Therefore, detailed sequence of steps for the solution of only SSDL-YY model is given in the following.

- Read system data and assign an initial approximate solution. If better solution estimate is not available, use the slack-start.
- Form nodal admittance matrix, and Initialize iteration count  $ITRP = ITRQ = r = 0$
- Compute Cosine and Sine of nodal rotation angles using (17) and (18), and store them. If they, respectively, are less than the Cosine and Sine of -48 degrees, equate them, respectively, to those of -48 degrees.
- Form  $(m+k) \times (m+k)$  size matrices  $[Y\theta]$  and  $[YV]$  of (1) and (2) respectively each in a compact storage exploiting sparsity. The matrices are formed using (22), (23), (11), and (24). In  $[YV]$  matrix, replace diagonal elements corresponding to PV-nodes by very large value (say,  $10.0^{10}$ ). Factorize  $[Y\theta]$  and  $[YV]$  using the same ordering of nodes regardless of node-types and store them using the same indexing and addressing information.
- Compute residues  $[\Delta P]$  at PQ- and PV-nodes and  $[\Delta Q]$  at only PQ-nodes. If all are less than the tolerance ( $\epsilon$ ), proceed to step-n. Otherwise follow the next step.
- Compute the vector of modified residues  $[RP]$  using (20) for PQ-nodes, and using (9) and (19) for PV-nodes.
- Solve (3) for  $[\Delta\theta]$  and perform an update,  $[\theta] = [\theta] + [\Delta\theta]$ .
- Set voltage magnitudes of PV-nodes equal to the specified values, and Increment the iteration count  $ITRP = ITRP + 1$  and  $r = (ITRP + ITRQ)/2$ .
- Compute residues  $[\Delta P]$  at PQ- and PV-nodes and  $[\Delta Q]$  at PQ-nodes only. If all are less than the tolerance ( $\epsilon$ ), proceed to step-n. Otherwise follow the next step.
- Compute the vector of modified residues  $[RQ]$  using (21) for only PQ-nodes.
- Solve (5) for  $[\Delta V]$  and update PQ-node voltage magnitudes using  $[V] = [V] + [\Delta V]$ . While solving (5), skip all the rows and columns corresponding to PV-nodes.
- Calculate reactive power generation at PV-nodes and tap positions of tap changing transformers. If the maximum and minimum reactive power generation capability and transformer tap position limits are violated, implement the violated physical limits and adjust the loadflow solution.
- Increment the iteration count  $ITRQ = ITRQ + 1$  and  $r = (ITRP + ITRQ)/2$ , and Proceed to step-c.
- From calculated and known values of voltage magnitude and voltage angle at different power network nodes, and



tap position of tap changing transformers, calculate power flows through power network components, reactive power generation at PV-nodes, and real and reactive power generation at the slack-node.

#### IV. TEST RESULTS

The proposed method is tested with a number of well-ill-conditioned sample systems. It is also tested for the convergence sensitivity to branch R/X ratios. Two series of tests are generated, one by lowering the base case branch reactance and the other by raising the base branch resistances. Both scaling of a parameter of each of the branches simultaneously (uniform scaling) and scaling of a parameter of only one branch at a time (non-uniform scaling) are used. The latter generates more realistic difficult cases of a wide R/X ratio disparity among branches incident to a node. Wide disparities are also created at the terminal nodes of branches of zero resistances with the uniform scaling.

When an increased R/X ratio is created by lowering reactance and leaving resistance the same, the branch impedance decreases, and improves voltage regulation for the receiving terminal node of a branch. The situation can be considered to be dominantly simulating R/X ratio test condition [6]. A similar R/X ratio can be created by raising the branch resistance and leaving reactance unchanged. This increases the branch impedance and causes poor voltage regulation for the receiving terminal node of a branch. For a five-node network under uniform R-scaling test condition of  $(7.1R+jX)$ , the minimum voltage magnitude in the solution is found to be about 0.5 p.u. and the system is very close to its voltage stability limit. Larger scaling factor caused divergence of the solution process. Such test conditions can be said to have simulated system non-linearity, which dominantly influence the convergence.

All the tests are performed in single precision on personal computer using slack-start initialization. Results of the tests are given in terms of the number of iterations required to converge to 0.01 Mw/Mvar mismatch  $\Delta P_r/\Delta Q_r$ , unless another specific tolerance is mentioned.

Programming of the polar co-ordinate formulations of the loadflow methods studied implements rectangular power mismatch calculations involving storage of voltage magnitude vector and evaluation of trigonometric functions, sine and cosine of  $\Delta\theta$ , in the iteration process [9]. Results are provided for trigonometric functions evaluation in all iterations. However, studies on different systems have shown that it is sufficient to evaluate trigonometric functions only in the first iteration with  $\Delta\theta$  assumed zero in subsequent iterations.

Since off diagonal elements are made of branch susceptances and admittance magnitudes in SSDL-YY method, calculation of gain matrices involve taking only a square-root for each branch except those connected between two PV-nodes and a PV-node and the slack-node, and each diagonal element takes only 4-multiplication, a division, and a subtraction operation for  $I=1$  at which value all the method of this paper gives the best possible convergence. Also it takes only two multiplications and an addition operation more than the GFDL method at a PQ-node in each iteration. This overhead of super decoupling is negligible and can be ignored. Therefore, only iteration counts are used as the basis of

comparison of proposed SSDL-YY method with the SSDL-BX and GFDL methods.

Presently the only reasonable comparison in this paper is that between the GFDL method, its advanced version manifested as SSDL-BX method, proposed SSDL-YY method and FSDL [6] method. Results of Tables-1, Table-2, Table-3, Table-5 and Table-6 provide this comparison. Clearly the SSDL-YY method is to be recommended. The comparison can be summarized as follows:

- (a) Relatively better performance of all the methods presented in this paper could be achieved for values of  $I=0, 1, 2$ . Other values of  $I$  caused increased iterations to obtain converged solution. The best performance could be obtained for  $I=1$  for all of the methods, and therefore, test results of SSDL-BX and the proposed SSDL-YY methods are provided only for  $I=1$ .
- (b) Table-1 gives the convergence results for GFDL, SSDL-BX, SSDL-YY, and FSDL methods under uniform R-scale and X-scale test conditions. The comparisons in (c), (d), (e) and (f) are based on results of Table-1.
- (c) The SSDL-BX method takes maximum of 6-5, 2-1, 4-4, 9-8, and 1-1 iterations less than the GFDL method for 5-, 14-, 26-, 57-, and 118-node systems, respectively, under uniform R-scaling test conditions, whereas it takes maximum of only 1-1 and 0-1 iterations more for 14-node and 118-node systems respectively. SSDL-BX converges better for all systems tested over complete range of uniform X-scaling test conditions. Moreover it can be seen from Table-1 that SSDL-BX method converges in 6-instances when the GFDL method fails to provide solution, whereas it fails only in two instances when GFDL method provides solution. Therefore, SSDL-BX method is more efficient and reliable than GFDL method. This fact is further established from the analysis of the number of test case given in Table-3.
- (d) The factor  $K_p$  as determined in (19) is restricted to 0.75 to begin with, and it is changed to 0.6 when the average of all the less than 1.0 value at PV-nodes is less than 0.6 for all versions of SSDL method in this paper. This two level restriction to  $K_p$  is also applied to FSDL method. It can be seen that thus modified FSDL method tested for this paper itself is the substantial improvement over originally presented by this author in [6] particularly for 57-node system and under uniform X-scale factors 0.1 and less. Now, from the results of Table-1, it can seen that SSDL-YY method takes maximum of 1-1, 1-1, 3-4, 1-2, and 1-1 iterations less than FSDL method for 5-, 14-, 26-, 57- and 118-node systems respectively under uniform R-scaling test conditions. The only exception is the uniform R-scaling test condition  $(3.5R+jX)$  where SSDL-YY method takes 1-1 iteration more in case of 14-node system.
- (e) SSDL-YY method takes maximum of 1-1, 1-2, 3-4, 1-1, 0-1 or 1-0 iterations less than FSDL method for 5-, 14-, 26-, 57-, and 118-node systems respectively under uniform X-scaling test conditions for scale factors greater than 0.2, whereas it takes only 1-1 iteration more in case of 57-node system. For scale factors less than 0.2, SSDL-YY method takes maximum of 2-2, 3-4, and 7-7 iterations less for 5-, 26-, and 118-node systems respectively, where-

Table-1: Iterations for convergence to 0.01 Mw/Mvar (Uniform scale factors)

Test Factor	5-node System [10]			14-node System [11]			26-node system [12]			57-node System [11]			118-node System		
	GFDL	SSDL	FSDL	GFDL	SSDL	FSDL	GFDL	SSDL	FSDL	GFDL	SSDL	FSDL	GFDL	SSDL	FSDL
	BX	YY		BX	YY		BX	YY		BX	YY		BX	YY	
Increased non-linearity condition tests generated by application of uniform R-scale factors															
0.5R+jX	4-3	3-2	3-2	4-3	4-3	3-3	3-3	4-3	8-8	4-4	5-4	8-8	6-6	7-6	7-6
R+jX	4-4	3-3	3-3	4-3	5-4	4-3	4-4	8-8	4-4	5-4	8-8	7-7	7-6	7-6	7-6
1.5R+jX	4-3	4-3	4-3	4-3	6-5	4-4	4-4	8-8	5-5	6-5	9-9	9-9	7-6	7-6	8-7
2R+jX	5-4	4-3	4-3	4-4	6-5	5-4	5-5	8-8	6-5	6-5	9-9	12-11	8-7	8-7	8-8
2.5R+jX	5-4	4-4	4-3	4-4	6-5	6-5	5-5	9-8	6-5	6-5	9-9	14-14	10-9	9-8	9-9
3R+jX	6-6	5-4	4-4	4-4	7-6	7-6	7-6	10-9	7-6	8-7	9-8	18-17	12-11	10-9	11-11
3.5R+jX	8-7	5-4	4-4	5-4	8-7	9-8	9-8	12-12	12-12	14-13	16-15	23-22	14-14	12-11	13-12
4R+jX	9-8	6-5	4-3	5-4	13-12	14-13	13-12	nc	nc	nc	nc	26-25	nc	18-17	18-17
4.5R+jX	9-8	6-5	4-4	5-5	nc	nc	nc	nc	nc	nc	nc	nc	nc	nc	nc
5R+jX	10-9	6-5	5-4	5-5											
5.5R+jX	11-10	7-6	5-5	6-6											
6R+jX	13-13	8-7	6-6	6-6											
6.5R+jX	16-15	10-10	7-7	7-7											
7R+jX	nc	nc	13-13	13-13											
7.5R+jX			nc	nc											
Increased branch R/X ratio tests generated by application of uniform X-scale factors															
R+jX	4-4	3-3	3-3	4-3	5-4	4-3	4-3	4-4	8-8	4-4	5-4	8-8	7-7	7-6	7-6
R+j.9X	4-4	4-3	3-3	4-3	5-5	4-3	4-4	5-4	8-8	4-4	5-5	8-8	7-7	6-6	6-5
R+j.8X	4-4	4-3	3-2	4-3	6-5	4-3	4-4	5-4	8-8	5-4	5-5	8-8	7-7	6-5	6-5
R+j.7X	4-4	4-3	3-2	4-3	6-5	4-3	5-4	5-5	7-7	5-4	6-5	8-8	8-7	6-5	6-5
R+j.6X	5-4	4-3	3-2	4-3	6-5	4-3	5-5	6-5	7-7	5-5	6-5	7-7	8-8	6-5	6-5
R+j.5X	5-4	4-3	3-2	4-3	7-6	4-3	6-6	7-6	7-7	6-5	6-5	7-7	9-8	6-6	7-7
R+j.4X	5-5	4-3	3-2	4-3	8-7	4-3	6-6	7-6	7-6	6-5	6-5	7-7	9-8	7-6	7-7
R+j.3X	6-5	4-3	3-2	4-3	8-7	5-4	6-5	6-5	8-7	6-6	6-5	7-6	10-10	9-8	8-7
R+j.2X	7-6	4-3	4-3	4-4	9-8	5-5	6-5	7-7	9-9	7-6	7-6	7-7	13-12	12-11	10-9
R+j.1X	9-9	4-3	5-4	6-5	12-11	8-7	9-8	8-8	12-11	8-7	13-12	15-15	30-29	nc	14-13
R+j.09X	10-9	4-3	5-5	6-5	12-11	9-8	9-8	8-8	13-12	8-8	15-14	18-18	nc	18-17	16-15
R+j.08X	12-11	4-3	5-5	6-5	12-11	9-8	10-9	9-8	20-19	10-9	16-16	15-15		24-23	22-21
R+j.07X	11-10	4-3	5-5	7-6	13-12	10-10	10-10	9-9	nc	15-15	23-22	22-21		34-33	34-33
R+j.06X	11-10	5-4	6-5	7-7	21-21	14-13	12-11	9-9	nc	nc	nc			nc	nc
R+j.05X	13-12	5-4	6-5	7-7	nc	nc	26-25	21-20							
R+j.04X	nc	6-5	6-5	7-7			nc	nc							
R+j.03X		7-6	7-6	8-7											
R+j.02X		6-5	7-7	9-8											
R+j.01X		nc	7-7	9-9											

nc: Not converged in 100 iterations or diverged

as it takes maximum of 5-5, 1-1, 2-2, and 2-2 iterations more for 14-, 26-, 57-, and 118-nodes systems.

- (f) From (d) and (c) in the above it can be said that SSDL-YY method is overall better convergent than modified FSDL method tested for this paper, which in itself is an improvement over the one originally presented in [6].
- (g) Table-2 gives average number of iterations. Many loadflows are run for each non-uniform scale factor. Loadflow cases are generated by applying the same factor one at a time to different branches of a system. The average for 5-node system are over 7-cases of 7-branches for each scale factor. The averages are over 32-cases of 32-branches for 26-node system, and they are over 10-cases of first 10-lines for 118-node system. For each of the systems studied, individual R/X ratios are raised by the highest factor of 20 for non-uniform R-scaling and lowest factor of 0.03 for non-uniform X-scaling. It can be seen that a reduction in the average iterations is achieved for the SSDL-YY method compared to both SSDL-BX and FSDL methods. Average iterations for GFDL method are not given as it is established in (a) above that SSDL-BX method is better convergent and more reliable than

GFDL method. The reduction can be as much as about 230%, 124%, and 87% against SSDL-BX method under non-uniform R-scaling test conditions, and 320%, 48%, and 130% under non-uniform X-scaling test conditions for 5-, 26-, and 118-node systems respectively. Moreover, the reductions in average iterations for SSDL-YY method compared to FSDL method are about 12%, 80%, and 14% under non-uniform R-scaling test conditions, and about 13%, 80%, and 14% under non-uniform X-scaling test conditions for 5-, 26-, and 118-node systems respectively. However, slight increase of about 3% and 5% in average iterations by SSDL-YY against FSDL method is observed under non-uniform X-scaling test conditions for 5-, and 118- node systems.

- (h) Table-3 gives the analysis of total number of cases used to produce results of Table-2 for different test power systems. For each branch, seven non-uniform R- and X-scale factors other than the base case of Table-2 are considered. Thus the total number of cases are 49, 140, 224, 287, 70 and 70 for 5-node system of 7-branches, 14-node system of 20-branches, 26-node system of 32- branches, 30-node system of 41-branches, first 10-branches in 57-

Table-2: Average iterations for convergence to 0.01 Mw/Mvar (Non-uniform scale factors)

Test conditions	Number of system nodes								
	SSDL-BX	5-node FSDL	SSDL-YY	SSDL-BX	26-node FSDL	SSDL-YY	SSDL-BX	118-node FSDL	SSDL-YY
R+jX	3.00-3.00	4.00-3.00	3.00-3.00	4.00-4.00	8.00-8.00	5.00-4.00	4.00-4.00	4.00-4.00	4.00-3.00
5R+jX	5.29-4.71	4.29-3.86	4.00-3.43	4.88-4.50	8.19-8.19	5.13-4.38	5.00-4.30	5.10-4.40	4.90-4.50
10R+jX	8.86-8.29	4.71-4.43	4.57-4.14	6.56-6.31	8.34-8.34	5.44-4.78	6.80-6.30	5.10-4.70	5.10-4.70
15R+jX	11.57-11.00	4.71-4.57	4.43-4.00	8.41-8.19	8.56-8.53	5.72-4.94	7.80-7.50	5.30-4.80	5.00-4.60
20R+jX	13.71-13.57	5.00-4.57	4.43-4.14	11.78-11.63	8.88-8.81	5.91-5.19	8.90-8.60	5.10-4.90	4.90-4.60
R+jX	3.00-3.00	4.00-3.00	3.00-3.00	4.00-4.00	8.00-8.00	5.00-4.00	4.00-4.00	4.00-4.00	4.00-3.00
R+j0.1X	9.14-8.29	5.00-4.14	5.29-4.57	5.75-5.28	8.38-8.38	6.00-5.34	8.40-7.60	6.70-6.00	6.80-6.30
R+j0.05X	16.14-15.29	6.00-5.57	6.00-5.57	8.00-7.59	8.63-8.44	6.72-6.03	13.50-12.60	8.40-8.30	8.00-7.40
R+j0.03X	26.14-25.29	6.43-5.86	6.57-6.00	10.81-10.31	9.31-9.00	7.31-6.63	20.10-19.30	9.90-9.40	9.10-8.40

node system, and first 10-branches in 118-node system respectively. The results clearly indicate that the convergence performance and the reliability of obtaining solution is the best for the SSDL-YY method among the methods tested for this paper, followed by FSDL, and SSDL-BX methods. This fact is also established from the results of Table-1.

Table-3: Convergence results for cases of non-uniform scaling

Method	System Number Nodes	Number of cases solved in N iterations where N is						No. Of DIV Cases
		N≤5	5<N≤10	10<N≤20	20<N≤50	N>50		
GFDL	5	5	22	14	8	0	0	49
	14	44	43	38	10	2	3	140
	26	0	170	45	5	1	3	224
	30	63	94	95	24	2	9	287
	57	0	50	14	6	0	0	70
	118	11	35	17	7	0	0	70
SSDL-BX	5	5	21	15	8	0	0	49
	14	44	44	37	10	2	3	140
	26	109	68	41	4	2	0	224
	30	71	86	95	25	2	8	287
	57	0	52	13	5	0	0	70
	118	12	35	19	4	0	0	70
FSDL	5	29	20	0	0	0	0	49
	14	105	30	1	0	1	3	140
	26	0	209	15	0	0	0	224
	30	212	67	3	0	0	5	287
	57	0	63	4	1	0	0	70
	118	28	36	6	0	0	0	70
SSDL-YY	5	30	19	0	0	0	0	49
	14	106	29	1	0	1	3	140
	26	139	77	8	0	0	0	224
	30	230	49	3	0	0	5	287
	57	0	67	2	1	0	0	70
	118	32	32	6	0	0	0	70

- (i) Table-4 gives number of iterations taken by the SSDL-YY method for different power mismatch requirements under three uniform scaling test conditions including the base-case of (R+jX). The number of iterations rises smoothly in line with the tolerance.
- (j) Table-5 gives number of iterations taken by different method for 13-node and 43-node ill-conditioned systems. It should be noted that better convergence by SSDL-YY

Table-4: Iterations by SSDL-YY for different tolerances

Uniform Scaling test Condition	Tolerance in Mw/Mvar	Number of system nodes				
		5	14	30	57	118
2R + jX	1.000	2-1	3-3	3-2	5-4	3-2
	0.100	3-2	4-4	4-4	6-5	4-3
	0.010	4-3	5-5	5-4	8-7	5-5
R + jX	1.000	2-1	2-2	3-2	4-3	3-2
	0.100	3-2	3-2	3-2	5-4	3-3
	0.010	3-3	4-3	4-3	7-6	4-3
R + j0.2X	1.000	2-1	4-3	4-3	5-4	5-4
	0.100	3-2	5-4	5-5	7-6	7-6
	0.010	4-3	6-5	8-7	10-9	9-9

method could be achieved by tuning rotation angle  $\Phi_p$  to  $-0^\circ$  for both the systems. The better convergence by FSDL method could be achieved by tuning  $\Phi_p$  to  $-0^\circ$  for 13-node system and by tuning  $\Phi_p$  to  $-90^\circ$  for 43-node system.

Table-5: Iterations for convergence to 0.01 Mw/Mvar

System No. of Nodes	Method							
	SSDL-YY			FSDL			SSDL-BX	GFDL
	-48°	-0°	-90°	-36°	-0°	-90°		
13 [13]	9-9	5-4	24-24	6-5	5-4	20-19	6-5	4-4
43 [13]	20-19	15-14	20-19	15-14	29-28	12-11	14-14	18-17

- (k) Table-6 gives number of iterations taken by different methods to converge for each of the 2-small, and 6-radial distribution systems. It can be seen that SSDL-YY method always converge in less number of iterations than those by FSDL method. The SSDL-BX method also converges in less number of iterations than by GFDL method in all except 40-node system in which case it takes almost double the iterations. GFDL method diverges in case of 31-node system indicated by double star (\*\*). However, SSDL-YY method converges in comparable to substantially less iterations than SSDL-BX method. Therefore, again SSDL-YY method turns out to be the winner among all.



Table-6: Iterations for convergence to 0.01 Mw/Mvar								
METHOD	Number of System Nodes							
	3 [14]	11 [14]	13 [15]	15 [17]	22 [15]	31 [16]	40 [16]	85 [17]
SSDL-YY	5-4	6-5	4-4	2-1	5-4	7-7	7-7	4-3
FSDL	7-7	7-6	5-4	3-2	8-7	9-8	9-8	4-4
SSDL-BX	5-4	6-5	4-3	3-2	5-4	26-25	28-27	5-4
GFDL	7-7	7-6	4-4	4-3	7-6	**	14-13	7-6

## V. FURTHER RESEARCH DIRECTION

There are two further research directions. One is to study convergence pattern in the iterative solution process of the proposed method, and develop acceleration factors that can produce solution in maximum of 2 to 3 iterations. The other is to use uniform and non-uniform P-scale and Q-scale factors for real power and reactive power loads in addition to uniform and non-uniform R-scale and X-scale factors in training Artificial Neural Network (ANN) using the proposed method to achieve very fast and almost closed form of solution.

## VI. CONCLUSION

A generalized SSDL-X'X' loadflow method is presented leading to SSDL-BX and the proposed SSDL-YY methods. The proposed method is characterized by gain matrices defined independent of rotation angles, rotation angles restricted to maximum of  $-48^\circ$ , modification of real power mismatches at PV-nodes to correct for model inaccuracy, an efficient slack-start as initial guess solution, and the presence of transformed specified reactive power in the diagonal elements of the gain matrix [YV] of the Q-V sub-problem.

The proposed SSDL-YY method is the most efficient and reliable among all the decoupled loadflow methods. The convergence of the method is demonstrated to be better than that of the SSDL-BX method for all normal R/X ratio systems to those having high R/X ratio branches. For systems of large R/X ratio branches, the proposed method can be more than twice faster than the SSDL-BX method, which is more reliable and better convergent than the GFDL method. It is also found to be better convergent than the FSDL method.

Attractive features of the proposed method are that it is not very sensitive to the restriction applied to the rotation/transformation angle, and it provides an elegant formulation for the diagonal elements of the gain matrix [YV] that suggest a mechanism for their numerical value manipulation particularly useful when diagonal dominance issue arise in the presence of a capacitive series branch or excessive capacitive compensation at a node.

Existing decoupled loadflow programs can be easily upgraded to the SSDL-YY as is apparent from the description of the method. It is believed that all applications of the decoupled loadflow methods would benefit from the proposed method, and it would potentially be the preferred general purpose loadflow method.

## VII. APPENDIX

A. Active and reactive power mismatch equations divided by  $V_p^1$  on both sides and partial derivatives

$$\Delta P_p/V_p^1 = P_p^{sh}/V_p^1 - (G_{pp} + g_p)/V_p^{(1-2)} - \sum_{q \neq p} (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) V_q/V_p^{(1-1)} \quad (32)$$

$$\Delta Q_p/V_p^1 = Q_p^{sh}/V_p^1 + (B_{pp} + b_p)/V_p^{(1-2)} - \sum_{q \neq p} (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) V_q/V_p^{(1-1)} \quad (33)$$

$$-\partial(\Delta P_p/V_p^1)/\partial \theta_p = \sum_{q \neq p} (-G_{pq} \sin \theta_{pq} + B_{pq} \cos \theta_{pq}) V_q/V_p^{(1-1)} \quad (34)$$

$$-\partial(\Delta Q_p/V_p^1)/\partial \theta_p = \sum_{q \neq p} (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) V_q/V_p^{(1-1)} \quad (35)$$

$$-\partial(\Delta P_p/V_p^1)/\partial \theta_q = (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) V_q/V_p^{(1-1)} \quad (36)$$

$$-\partial(\Delta Q_p/V_p^1)/\partial \theta_q = -(G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) V_q/V_p^{(1-1)} \quad (37)$$

$$-\partial(\Delta P_p/V_p^1)/\partial V_p = IP_p^{sh}/V_p^{(1+1)} - (1-2)(G_{pp} + g_p)/V_p^{(1-1)} - (1-1) \sum_{q \neq p} (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) V_q/V_p^1 \quad (38)$$

$$-\partial(\Delta Q_p/V_p^1)/\partial V_p = IQ_p^{sh}/V_p^{(1+1)} + (1-2)(B_{pp} + b_p)/V_p^{(1-1)} - (1-1) \sum_{q \neq p} (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) V_q/V_p^1 \quad (39)$$

$$-\partial(\Delta P_p/V_p^1)/\partial V_q = (G_{pq} \cos \theta_{pq} + B_{pq} \sin \theta_{pq}) V_p/V_q^{(1-1)} \quad (40)$$

$$-\partial(\Delta Q_p/V_p^1)/\partial V_q = (G_{pq} \sin \theta_{pq} - B_{pq} \cos \theta_{pq}) V_p/V_q^{(1-1)} \quad (41)$$

In the fixed Jacobian methods, the Jacobian is to be determined only once at the start with an initial guess. Considering the guess solution to be the slack-start ( $V_s \angle \theta_s$  for all the nodes), the expressions for the matrix elements reduce to the following:

$$-\partial(\Delta P_p/V_p^1)/\partial \theta_p = -B_{pp}/V_s^{(1-2)} \quad -\partial(\Delta Q_p/V_p^1)/\partial \theta_p = -G_{pp}/V_s^{(1-2)}$$

$$-\partial(\Delta P_p/V_p^1)/\partial V_p = [IP_p^{sh}/V_s^2 + (G_{pp} - (1-2)g_p)]/V_s^{(1-1)}$$

$$-\partial(\Delta Q_p/V_p^1)/\partial V_p = [IQ_p^{sh}/V_s^2 - (B_{pp} - (1-2)b_p)]/V_s^{(1-1)}$$

$$-\partial(\Delta P_p/V_p^1)/\partial \theta_q = -B_{pq}/V_s^{(1-2)} \quad -\partial(\Delta P_p/V_p^1)/\partial V_q = G_{pq}/V_p^{(1-1)}$$

$$-\partial(\Delta Q_p/V_p^1)/\partial \theta_q = -G_{pq}/V_s^{(1-2)} \quad -\partial(\Delta Q_p/V_p^1)/\partial V_q = -B_{pq}/V_p^{(1-1)}$$

Where,  $I = -\infty, \dots, -2, -1, 0, 1, 2, \dots, +\infty$ .

The elements of the transformed gain matrices [Y0] and [YV] determined from the generalized Jacobian elements given in the above remain the same for any value of "T" except diagonal elements of the gain matrix [YV] of the Q-V sub-problem. The parameter  $h_p$  that appears in the diagonal element of the gain matrix [YV] can be expressed by (12) for different value of the raised power "T" for the generalized model SSDL-X'X', and it is expressed by (24) for the proposed SSDL-YY model.

The equations (12) and (24) provide elegant formulation for the diagonal elements of the gain matrix [YV] that suggest a mechanism for their numerical value manipulation particularly useful when diagonal dominance issue arise in the presence of capacitive series branch or excessive capacitive compensation at a node. It is always desirable to obtain a solution even if it is computationally expensive against not to have one.

B. Transformed Power Flow Equations

Real and imaginary parts of transformed power flow equations from [4] and [6] are given as:

$$P_p' = V_p'^2(G_{pp}' + g_p') + V_p \sum_{q \neq p} (G_{pq}' \cos \theta_{pq} + B_{pq}' \sin \theta_{pq}) V_q \quad (42)$$

$$Q_p' = -V_p'^2(B_{pp}' + b_p') + V_p \sum_{q \neq p} (G_{pq}' \sin \theta_{pq} - B_{pq}' \cos \theta_{pq}) V_q \quad (43)$$

$$\text{where, } G_{pq}' + jB_{pq}' = Y_{pq} \exp(j(\Phi_{pq} + 180^\circ + \Phi_p)) \quad (44)$$

$$G_{pp}' + jB_{pp}' = Y_{pp} \exp(j(\Phi_{pp} + \Phi_p)) \quad (45)$$

$$g_p' + jb_p' = y_p \exp(j(\Phi_{pb} + \Phi_p)) \quad (46)$$

### C. Transformation of Branch Admittance

The gain matrices in their transformed version are asymmetrical because mostly different rotations are required to be applied at the terminal nodes of a branch [4]. However, symmetrical gain matrices can be obtained by the Haley and Ayres technique [5] of applying an average of rotations at the terminal nodes of a branch to the branch admittance, and it involves calculation of an inverse of the Tangent at each node to know the rotation angle and the Sine as well as Cosine trigonometric functions of the average of the rotations at the terminal nodes of a branch.

The constant symmetrical gain matrices of the Decoupled Loadflow Models SSDL-YY, SSDL-XX, and SSDL-BX are independent of rotation angles. However, gain matrices of the SSDL-X'X' and other models described in [18] are dependent on rotation angles as their elements are determined by transforming a branch admittance as shown in following steps in order to keep the gain matrices symmetrical.

$$1. \text{ Compute: } \Phi_p = \arctan(G_{pp}/B_{pp}) \text{ \& } \Phi_q = \arctan(G_{qq}/B_{qq}) \quad (47)$$

$$2. \text{ Compute the average of rotations at the terminal nodes (p and q) of a branch: } \Phi_{av} = (\Phi_p + \Phi_q)/2 \quad (48)$$

$$3. \text{ Compare } \Phi_{av} \text{ with the Limiting Rotation Angle (LRA) and let } \Phi_{av} \text{ to be the smaller of the two:} \\ \Phi_{av} = \text{minimum}(\Phi_{av}, \text{LRA}) \quad (49)$$

$$4. \text{ Compute transformed pq-th element of the admittance matrix: } G_{pq}' + jB_{pq}' = (\cos \Phi_{av} + j \sin \Phi_{av})(G_{pq} + jB_{pq}) \quad (50)$$

5. Note that the transformed branch reactance is:

$$X_{pq}' = B_{pq}' / (G_{pq}'^2 + B_{pq}'^2) \text{ and similarly,} \quad (51)$$

$$X_{pp}' = B_{pp}' / (G_{pp}'^2 + B_{pp}'^2) \quad (52)$$

In the description above  $X_{pq}'$  is the transformed branch reactance defined by equation (51) and  $B_{pq}'$  is the corresponding transformed element of the susceptance matrix.  $G_{pp}'$  and  $B_{pp}'$  are diagonal elements as given in (52).

### D. Loadflow Models organized for Global Corrections

The loadflow models described in this paper and all others can be organized to produce corrections to the initial estimate solution or global corrections in all iterations. This involves storage of the vectors of modified residues and replacing the equations (7), (8), (9), (30), (4) and (6) respectively by (53), (54), (55), (56), (57) and (58). Superscript '0' in equations (57) and (58) indicates the initial solution estimate.

$$RP_p' = [(\Delta P_p')' + (G_{pp}'/B_{pp}')(\Delta Q_p')'] / (V_p^0)^2 + RP_p^{(n-1)} \quad (53)$$

$$RQ_p' = [(\Delta Q_p')' + (G_{pp}'/B_{pp}')(\Delta P_p')'] / V_p^0 + RQ_p^{(n-1)} \quad (54)$$

$$RP_p' = \Delta P_p' / (K_p V_p^0) + RP_p^{(n-1)} \quad (55)$$

$$RP_p' = \Delta P_p' / V_p^0 + RP_p^{(n-1)} \quad (56)$$

$$\theta_p' = \theta_p^0 + \Delta \theta_p' \quad (57)$$

$$V_p' = V_p^0 + \Delta V_p' \quad (58)$$

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